Math3C: Quiz 2

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Note that these answers are not reviewed by Dr. Weisbert and/or may include some errors (as we figured out first week). If you find one or more, let me know and I'll edit the document. My email is wesleytk@ucla.edu.

C Level Problem. How many words (nonsense words included) can you make using all of the letters in the word AABBBCCCCD? An example of such a word is BAABBCCDCC.

You should know this. The length of the word is 10. There as 2 A's, 3 B's, 4 C's and 1 D. That gives

$$\frac{10!}{2!3!4!1!}$$

B Level Problem. You are to set up a day of Summer camp for kids. You must plan activities for the day. The six possible activities to choose from are: swimming in the lake, ropes course, arts and crafts, group hike, archery and feed the grizzly bear. In one day, you will have the kids perform exactly one of the six activities exactly once, exactly one of the six activities exactly two times, and exactly one of the six activities exactly three times. No other activity will be performed. In how many ways can you schedule the day? Note that activities done in a different order count as different schedules.

Let's split this problem into two problems (because it's a B level, we're expecting that): (1) what activities we do (2) what order we do them in.

Part 1: What activities we do. We focus on the word *exactly.* That means that you can't choose the same activity for multiple things. That means that we have 6 choices for the first type of activity (say the one to do once), 5 choices for the next, and 4 choices for the last (say the one to do thrice). Overall, that's

$$6 \cdot 5 \cdot 4 = P(6,3) = \frac{6!}{3!}$$

Part 2: What order we do them in. This is the same as the A level problem, only the word is ABBCCC. So we get:

$$\frac{6!}{1!2!3!}$$

What do we do with the parts? It's a tuple, or, for each choice of three things to do, we have all the possible orders of those things, so we multiply. Final answer:

$$\frac{6!}{3!}\frac{6!}{2!3!}$$

A Level Problem. A class of 10 students contains identically dressed quadruplets (four people who appear identical). All other students have distinguishing features. You select six students, line them up, and take a photograph of the line of six students. How many photographs can you take?

This is best done by splitting into cases that each have 2 parts: (1) choose the people who aren't the quadruplets, (2) order the people you have. These parts are going to multiply within case, whereas we are going to add across cases.

Case 1: Select all 4 quadruplets, then you need $\binom{6}{2}$ other people. You finally have $\frac{6!}{4!}$ orders of those people you choose, with the quadruplets.

Case 2: Select 3/4 quadruplets, then you need $\binom{6}{3}$ other people. You have $\frac{6!}{3!}$ orders of those people you choose. Note that I'm ignoring how many choices of 3 of 4 quadruplets you have because you can't tell the difference between them in the final photo.

Case 3: Select 2/4 quadruplets, then you need $\binom{6}{4}$ other people. You have $\frac{6!}{2!}$ orders of those people you choose.

Case 4: Select 1/4 quadruplets, then you need $\binom{6}{5}$ other people. You have $\frac{6!}{1!}$ orders of those people you choose.

Case 5: Select 0/4 quadruplets, then you need $\binom{6}{6}$ other people. You have 6! orders of those people you choose.

These are partitions of the full set, or cases, so you should add to get:

$$\binom{6}{2}\frac{6!}{4!} + \binom{6}{3}\frac{6!}{3!} + \binom{6}{4}\frac{6!}{2!} + \left[\binom{6}{5} + 1\right] 6!$$

Note that I combined the last two cases because they were both multiplied by 6! alone.