

### Math3C: Quiz 3

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Note that these answers are not reviewed by Dr. Weisbert and/or may include some errors (as we figured out first week). If you find one or more, let me know and I'll edit the document. My email is wesleytk@ucla.edu.

**C Level Problem.** You roll a fair six sided die two times. What is the probability that the sum of the two rolls is 4?

Probability is:

$$P = \frac{\#win}{\#total}$$

Let's talk about winning first (getting the sum to be 4). How many ways are there to do that:

$$1 + 3$$

$$2 + 2$$

$$3 + 1$$

Okay, so there's 3 ways, and we rolled one die first, so order does truly matter. Now that tells us that order matters for the total too, giving us:  $6 \cdot 6$  total options. Our final answer:

$$\frac{3}{6^2}$$

**B Level Problem.** There are exactly 20 students in a class. You and two of your friends are in the class. A team of 5 people will be chosen at random from the class. What is the probability that you and your two friends will be chosen for the team?

Let's do the total counts first, because it's easier. You have 20 students, choose 5 of them:  $\binom{20}{5}$ .

Now let's figure out how many ways we can win. If you and your two friends are chosen, then there are only 2 more people to choose from the 17 that are not your friends, giving us:  $\binom{17}{2}$ . Final answer

$$\frac{\binom{17}{2}}{\binom{20}{5}}$$

**A Level Problem.** There are exactly 20 students in a class. You and four of your friends are in the class. A team of 5 people will be chosen at random from the class. What is the probability that you and at least three of your friends will be chosen for the team?

The at least is the annoying part of this problem. The total counts are the same:  $\binom{20}{5}$ .

Let's split this into cases:

- Case 1: You + 5 friends. This can't happen: the total team is bigger than 5.
- Case 2: You + 4 friends. By the same logic as the B level problem, we've already selected 5 of the 5 people in the team, so we're done. There's one way to do this because you've only got 4 friends.

- Case 3: You + 3 friends. By the same logic as the B level problem, we've already selected 4 of the 5 people in the team. We then have  $\binom{15}{1}$  choices to fill up the team because you have 15 people that aren't your friend in the class. But wait, what about selecting the 3 of 4 friends to join you on the team? The total counts each of those selections as different, so we need to as well. There are  $\binom{4}{3}$  ways to select the 3 of 4 friends to be with you. Giving a full case 3 count of:

$$\binom{15}{1} \binom{4}{3}$$

The total probability is then:

$$\frac{1 + \binom{15}{1} \binom{4}{3}}{\binom{20}{5}}$$