Math3C: Quiz 4
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Note that these answers are not reviewed by Dr. Weisbert and/or may include some errors (as we figured out first week). If you find one or more, let me know and I'll edit the document. My email is wesleytk@ucla.edu.

Suppose that a certain disease has a prevalence of $\frac{1}{4}$ in a population. Test A for this disease has a false positive rate of 8% and a false negative rate of 2%. Test B for this disease has a false positive rate of 1% and a false negative rate of 10%. The rates of false positives and false negativs for Test A and B depend only on whether or not the individual tested has the disease. Individuals are first tested with Test A and then tested with Test B.

C Level Problem. What is the probability that you have the disease, given that you test negative on Test A?

This is a classic Bayes formula question.

$$P(D|-_A) = \frac{P(-_A|D)P(D)}{P(-_A|D)P(D) + P(-_A|D^c)P(D^c)}$$

We get the denominator from the law of total probability for $P(-_A)$. Let's figure out all the values we need:

$$\begin{aligned}
P(D) =& \frac{1}{4} \\
P(D^c) =& \frac{3}{4} \\
P(-_A|D) =& 0.02 \\
P(+_A|D) =& 1 - P(-_A|D) = 0.98 \\
P(+_A|D^c) =& 0.08 \\
P(-_A|D^c) =& 1 - P(+_A|D^c) = 0.92
\end{aligned}$$

This gives us:

$$P(D|-A) = \frac{0.02 \cdot \frac{1}{4}}{0.02 \cdot \frac{1}{4} + 0.92 \cdot \frac{3}{4}}$$

B Level Problem. What is the probability that you have the disease given taht you test positive on Test A then negative on Test B?

This also is straight from the homework. (See my homework 5 on my website for more explanation.)

$$P(D|+_A -_B) = \frac{P(+_A|-_B, D)P(D, -_B)}{P(+_A -_B)} = \frac{P(+_A|D)P(-_B|D)P(D)}{P(+_A|D)P(-_B|D)P(D) + P(+_A|D^c)P(-_B|D^c)P(D^c)}$$

Now let's figure out all the values we need but don't have yet:

$$\begin{aligned}
P(-_B|D) =& 0.10 \\
P(+_B|D) =& 1 - P(-_A|D) = 0.90 \\
P(+_B|D^c) =& 0.01 \\
P(-_B|D^c) =& 1 - P(+_A|D^c) = 0.99
\end{aligned}$$

Now let's plug that in:

$$P(D|+_A -_B) = \frac{0.98 \cdot 0.10 \cdot \frac{1}{4}}{0.98 \cdot 0.10 \cdot \frac{1}{4} + 0.08 \cdot 0.99 \cdot \frac{3}{4}}$$

A Level Problem. What is the probability that you test positive on Test B given that you test positive on Test A?

The main confusion people had with this one is that it doesn't actually talk about disease status. That actually makes life easier. This is just a law of total probability problem. We just have:

$$\begin{aligned}
P(+_B|+_A) =& \frac{P(+_B \cap +_A)}{P(+_A)} = \frac{P(+_B \cap +_A|D)P(D) + P(+_B \cap +_A|D^c)P(D^c)}{P(+_A|D)P(D) + P(+_A|D^c)P(D^c)} \\
=& \frac{P(+_B|D)P(+_A|D) + P(+_B|D^c)P(+_A|D^c)}{P(+_A|D) + P(+_A|D^c)} \\
& \text{Plug and chug} \\
=& \frac{0.90 \cdot 0.98 \cdot \frac{1}{4} + 0.01 \cdot 0.08 \cdot \frac{3}{4}}{0.98 \cdot \frac{1}{4} + 0.08 \cdot \frac{3}{4}}
\end{aligned}$$